Applications of Temporal Conceptual Semantic Systems

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Abstract. Based on Formal Concept Analysis the notion of a Temporal Conceptual Semantic System is introduced as a formal conceptual representation for temporal systems with arbitrary discrete or continuous semantic scales. In this paper, we start with an example of a weather map with a moving high pressure zone to explain the basic notions for Temporal Conceptual Semantic Systems. The central philosophical notion of an object is represented as a formal concept or, more flexible, as a tuple of concepts. Generalizing the idea of a volume of an object in physics we introduce the notion of a *trace* of an object in some space. This space is described as a continuous or discrete concept lattice. Combining the notion of a trace of an object with the notion of a time granule yields the notion of a state of an object at some time granule. This general notion of a state allows for a clear conceptual understanding of particles, waves and Heisenberg's Uncertainty Relation. Besides these theoretical aspects, Temporal Conceptual Semantic Systems can be used very effectively in practice. That is shown for data of a distillation column using a nested transition diagram.

1 Introduction

In this paper we introduce a simple and powerful conceptual framework which is closely connected to many quite different fields of classical and modern knowledge representation. This framework has been developed in Conceptual Knowledge Processing which is based on Formal Concept Analysis [18, 10]. First ideas for this framework have been developed in applications of conceptual scaling in Fuzzy Theory [37, 38, 25], Rough Set Theory [14, 21], and General Systems Theory [5, 11, 12, 13, 20]. One of the basic intuitive notions in General Systems Theory is the notion of a state, but the formalization of this notion leads to many difficulties. We cite L. Zadeh [36]:

> *To define the notion of a state in a way which would make it applicable to all systems is a difficult, perhaps impossible, task.*

The notion of a state was defined in a first conceptual version by the author in [20] and generalized in several steps to the actual notion of a state in a Temporal Conceptual Semantic System in this paper. During this development a common generalization of the notion of states in physics [8, 6, 7, 27] and in automata theory [1, 9, 24] had been established. For a long time it was not clear how to define a conceptual notion of a *state* such that the quantum theoretical states which possess a probability distribution are also describable. The main step into that direction was the introduction of Conceptual Semantic Systems [26] which reached the aim to find a common conceptual description of particles and waves in physics.

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Based on the notion of *aspects* a first conceptual representation of Heisenberg's Uncertainty Relation has been presented in [29]. This has been developed further in [30] where a more general *object* notion, the *instance selection* and an explicit representation of *measurements* has been introduced. In the paper at hand the basic notions in Temporal Concept Analysis as developed by the author in previous articles [22, 23, 26, 27, 28, 29, 30] are generalized and clarified by the notion of a *Temporal Conceptual Semantic System* (TCSS). That is based on the notion of a *Conceptual Semantic System* (CSS) where two new tools are introduced, namely the formal representation of objects by *tuples of concepts* of the semantic scales and the *instance selection* which assigns to each tuple of concepts a "relevant" subset of the set of instances. In the following section we use a moving high pressure zone as a typical example of a *distributed* temporal object in a TCSS. We explain this special TCSS starting with a data table and interpreting the values of the data table as formal concepts of suitable formal contexts. For readers who are not familiar with the notions in Formal Concept Analysis [10] we introduce some formal contexts and their concept lattices in this example of a moving high pressure zone. Later on we will introduce the mathematical definitions around TCSSs.

2 Example: A Moving High Pressure Zone

As a small example we construct a TCSS which yields a weather map with a moving high pressure zone over Germany. To keep the data table small we construct a map of Germany using a coarse grid of longitude and latitude coordinates into which we embed 15 towns. To represent a moving high pressure zone we assume that the pressure has been measured in some weather stations (WS) at two consecutive days, say Monday and Tuesday. The data are shown in Tab.1 where the rows 1,...,15 show the latitude and longitude values of 15 German towns, the rows 16,...,25 show the pressure values (in hectopascal (hPa)) measured at certain days at certain weather stations located in some of the previously mentioned towns.

In Table 1 the row labeled by *instance* 1 tells that the *place Berlin* has *latitude* 52.5 and longitude 13.4 and no time and no pressure is recorded in this line, shown by the sign "/" in the column for *time* and *pressure*. *Instance* 16 tells that *Weather Station Dortmund* has *latitude* 51.5 and *longitude* 7.5 and has reported at *Monday* a *pressure* of 1020 hectopascal.

2.1 Basic Conceptual Notions

For the conceptual representation of knowledge as it is contained in Table 1 we interpret the values in each column as elements of a conceptual hierarchy which is formally represented as the concept lattice of a formal context. In Formal Concept Analysis [10] a formal context $\mathbb{K}$ is defined as a triple (G, M, I) of sets such that $I \subseteq G \times M$. The set G is called the set of formal objects ("Gegenstände"), M is called the set of formal attributes ("Merkmale"), and the binary relation I is called the incidence relation of the formal context (G, M, I). For $g \in G$ and $m \in M$ we write "gIm" instead of "$(g, m) \in I$". A formal concept of a formal context (G, M, I) is a pair (A, B) where $A \subseteq G$, $B \subseteq M$, and $A = \{g \in G \mid \forall_{m \in B} gIm\}$ and $B = \{m \in M \mid \forall_{g \in A} gIm\}$. The set A is called the *extent*, and B the *intent* of the formal concept (A, B). The set $\mathfrak{B}(\mathbb{K})$ of all formal concepts of a formal context $\mathbb{K}$ is ordered by: $(A, B) \leq (C, D) :\Leftrightarrow A \subseteq C (\Leftrightarrow B \supseteq D)$. The ordered set $(\mathfrak{B}(\mathbb{K}), \leq)$ is a complete lattice, called the *concept lattice* of $\mathbb{K}$. For each formal object $g \in G$ the smallest concept containing g in its extent is called the object concept $\gamma(g)$ of g. Dually the attribute concept $\mu(m)$ of a formal attribute m is defined. It can be proved easily that $\forall_{g \in G, m \in M} gIm \Leftrightarrow \gamma(g) \leq \mu(m)$.

Table 1: Data table of a Moving High Pressure Zone

instance	place	latitude	longitude	time	pressure
1	Berlin	52.5	13.4	/	/
2	Dortmund	51.5	7.5	/	/
3	Dresden	51.0	13.7	/	/
4	Frankfurt (Main)	50.1	8.7	/	/
5	Frankfurt (Oder)	52.3	14.5	/	/
6	Freiburg	48.0	7.9	/	/
7	Hamburg	53.6	10.0	/	/
8	Kassel	51.3	9.5	/	/
9	Köln	51.0	7.0	/	/
10	Magdeburg	52.1	11.6	/	/
11	München	48.1	11.6	/	/
12	Nürnberg	49.5	11.1	/	/
13	Passau	48.6	13.5	/	/
14	Saarbrücken	49.2	7.0	/	/
15	Wilhelmshaven	53.5	8.1	/	/
16	WS Dortmund	51.5	7.5	Monday	1020
17	WS Frankfurt (Main)	50.1	8.7	Monday	1030
18	WS Hamburg	53.6	10.0	Monday	970
19	WS Kassel	51.3	9.5	Monday	1010
20	WS München	48.1	11.6	Monday	980
21	WS Berlin	52.5	13.4	Tuesday	1020
22	WS Dresden	51.1	13.7	Tuesday	1010
23	WS Frankfurt (Oder)	52.3	14.5	Tuesday	1030
24	WS Hamburg	53.6	10.0	Tuesday	970
25	WS München	48.1	11.6	Tuesday	980

2.2 The scales of our example

In our example of a moving high pressure zone we represent the values in the column for time as formal concepts of the formal context $\mathbb{S}_{time}$, called the semantic scale for time, shown in Table 2.

Table 2: The time scale $\mathbb{S}_{time}$

$\mathbb{S}_{time}$	Monday	Tuesday	days
Monday	×		×
Tuesday		×	×
/			

In the formal context $\mathbb{S}_{time}$ the "missing value" "/" is treated as a formal object without any attribute. The concept lattice $\mathfrak{B}(\mathbb{S}_{time})$ is shown in Fig. 1. The arrow in Fig. 1 represents the time relation which is used to define transitions; that will be discussed later.

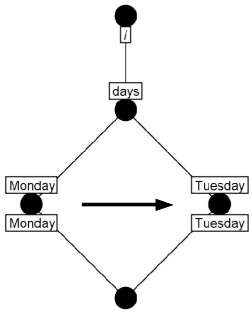

Fig. 1: The semantic scale for *time* with time relation

Similarly to the column for time we choose semantic scales for the other four columns of Table 1 which are different from the instance column; the instance column shows just a labeling of the rows.

The semantic scale $\mathbb{S}_p$ for *pressure* is defined as a modified interordinal scale on the multiples of 10 in the interval $[970, 1030]$. Its concept lattice is shown in Fig. 2.

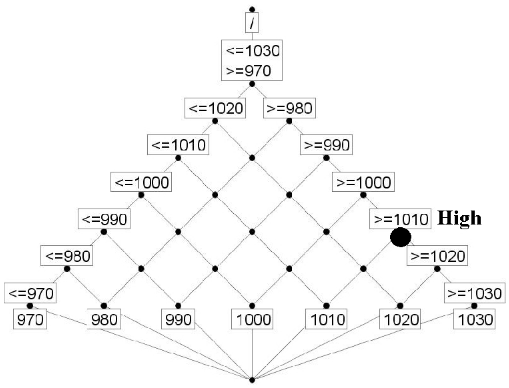

Fig. 2: The semantic scale for *pressure*

In this interordinal scale for pressure we choose the attribute concept $\mu(\geq 1010)$ for the representation of the notion of "high pressure", and call this formal concept **High**. Combined with the formal concept **Monday** in the time scale we like to form the tuple (**High, Monday**) as our formal representation of such an abstract "object" like "**the High at Monday**". To express formally that an abstract object like **High** can move over some landscape we have to connect the information about pressure and time with the places and their longitude and latitude values. For that purpose we use the "artificial key" of the instances. The theory for Temporal Conceptual Semantic Systems will be given in the next sections. As a result of that theory we show a visualization of the movement of the selected high pressure zone in Fig. 3.

Concerning the construction of the weather map in Fig. 3 we just mention the main steps:

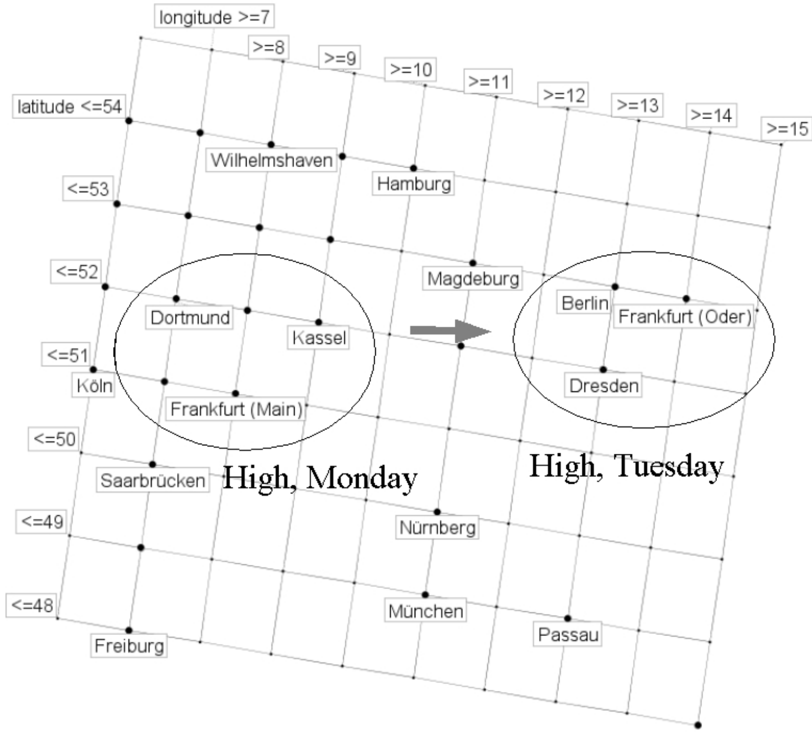

Fig. 3: A Weather Map with a Moving High Pressure Zone

- The construction of a grid of latitude-longitude-values as a semi-product of two ordinal scales for latitude and longitude.
- The construction of a map by embedding places using their latitude-longitude-values.
- The embedding of the traces of the chosen tuples of concepts, as for example the traces of the tuple (**High**, **Monday**) and (**High**, **Tuesday**) as visualized by ellipses in Fig. 3.
- The graphical representation of the time relation by arrows connecting the traces of the chosen tuples.

The details of the construction of the weather map in Fig. 3 can be seen in [31].

3 Conceptual Semantic Systems

3.1 Main ideas

For describing systems like for example physical or technical systems, social systems or the system of a family as represented by a psychological questionnaire we focus on system descriptions by data. We use Conceptual Semantic Systems as a simple and general tool for describing observations in reality. If there are regularities in a system, they can be described by dependencies among many-valued attributes, in the most simple case by functional dependencies.

In the following definition of a Conceptual Semantic System (CSS) we start with a family of formal contexts whose formal concepts are used as "basic semantic concepts" for the description of statements about some part of the reality. These statements are formally represented in the rows of a data table whose values are the chosen basic semantic concepts.

Remark: In the definition of a Conceptual Semantic System we do not explicitly represent the relational aspect of a statement as it is done in Conceptual Graphs [16, 17], in Power Context Families and Concept Graphs [19]. Recently, these relational structures have been combined with temporal CSSs by the author in Temporal Relational Semantic Systems [33, 34, 35].

In the data table of a CSS the rows are indexed by a set G which plays the role of an "artificial" key of the data table. G is also the set of formal objects of the semantically derived context which will be defined later. Our advice, that these formal objects should not be used in applications for the representation of objects in reality, yields the necessity of an alternative representation of objects in Conceptual Semantic Systems which will be given in the following sections.

3.2 Definition of a Conceptual Semantic System

Definition 1. ***"Conceptual Semantic System"***
Let M be a set, and for each $m \in M$ let $\mathbb{S}_m := (G_m,\ N_m,\ I_m)$ be a formal context and $\underline{\mathfrak{B}}(\mathbb{S}_m)$ its concept lattice. Let G be a set and

$$\lambda : G \times M \to \bigcup_{m \in M} \mathfrak{B}(\mathbb{S}_m)$$

be a mapping such that $\lambda(g, m) \in \mathfrak{B}(\mathbb{S}_m)$.

Then the quadruple

$$\mathfrak{K} := (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m \in M}, \lambda)$$

is called a Conceptual Semantic System (CSS) with semantic scales $(\mathbb{S}_m)_{m \in M}$. The elements of M are called many-valued attributes; the elements of G are called instances. We write $m(g) := \lambda(g, m)$ and $m(G) := \{m(g) \mid g \in G\}$.

We mention that the mapping λ can be interpreted as a (possibly infinite) data table whose values $m(g)$ "in column m" are formal concepts of the given semantic scale $\mathbb{S}_m$. For any instance g the tuple $(m(g)|m \in M)$ is interpreted as a short description of a statement connecting the concepts $m(g)$ where $m \in M$. A concept $m(g)$ may denote for example the grammatical subject, or the grammatical object, or the grammatical predicate of a statement. That allows for the representation of arbitrary, not only binary, relations; that has been made explicit in [35].

One of the main points in our intended interpretation of Conceptual Semantic Systems is that the instances are not interpreted as meaningful concepts, as for example objects like persons or particles. That differs strongly from the usual interpretation of many-valued contexts [10] where the formal objects are intended to represent objects in "reality". As formal objects they have to form a key in the data table of the many-valued context. In Conceptual Semantic Systems we are much more flexible in practical applications since we do not need an object domain whose objects form a key in the data table. Now we represent the information given in the 'data table' λ of a CSS by a single formal context, called the *semantically derived context* of the CSS.

Definition 2. ***"Semantically derived context"***
Let $\mathfrak{K} := (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ *be a CSS with semantic scales*

$$\mathbb{S}_m = (G_m, N_m, I_m)\ (m \in M),$$

*and let int(**c**) denote the intent of a concept **c**. Then the formal context*

$$\mathbb{K} := (G, N, J) \text{ where } N := \{(m,n) \mid m \in M, n \in N_m\} \text{ and}$$
$$gJ(m,n) :\Longleftrightarrow n \in int(m(g))$$

is called the semantically derived context of $\mathfrak{K}$.

We mention that a CSS $\mathfrak{K}$:= (G, M, $(\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ can be reconstructed from its semantically derived context $\mathbb{K} := (G, N, J)$ and the semantic scales $(\mathbb{S}_m)_{m\in M}$, since each value $(A, B) = \lambda(g, m)$ is uniquely determined by the intent of the object concept of g in the $m-$part $\mathbb{K}_m := (G, \{m\} \times N_m, J \cap (G \times (\{m\} \times N_m)))$ of $\mathbb{K}$. The corresponding "reconstruction property" for scaled many-valued contexts does not hold if there exists at least one scale $\mathbb{S}_m$ whose object concept mapping γ_m is not injective. The precise connection between the semantically derived context of a CSS and the derived context of a many-valued context is described in [31].

3.3 Object representation by tuples of semantic concepts

In the main interpretation of contexts and many-valued contexts the formal objects are used to represent certain "objects" in reality, like for example "living beings" in the formal context of Figure 1.1 in [10], p. 18. That led to difficulties in the formal representation of temporal systems where time granules, like for example hours or days, are used as formal objects. Clearly, in such data we do not want to represent other kinds of objects, like persons, also as formal objects. A very successful formal representation was the notion of an *actual object*, defined as a pair (p, t) of an object (for example a person) and a time granule; these actual objects had been chosen as formal objects of such "temporal" contexts. That led the author to the notion of a CTSOT (Conceptual Time Systems with actual Objects and a Time relation) [22, 23, 27] which are very well suited for the representation of life tracks of objects. The disadvantage of the CTSOTs is that *distributed objects* like a high pressure zone on a weather map can not be represented in a CTSOT since each actual object has, as a formal object, exactly one object concept in each part of the derived context - and is therefore not distributed.

That led the author to the notion of a CSS with the interpretation that the usually numerous kinds of general objects, like for example persons and places, should be treated in a symmetric way in the CSS. Hence none of these kinds of objects should play the special role of being represented by the formal objects. Therefore, we interpret the set of formal objects of a CSS as a "meaningless syntactical key" which can be understood as a set of labels for the rows of a data table. It is very advantageous in practical applications that we do not have to search for a kind of objects in practice which can be represented as a key of our intended data table.

In the following we represent objects, like for example persons, days or towns, as formal concepts of the chosen semantic scales of a CSS. Then we would also like to have a formal representation of concatenations of formal concepts of different semantic scales as for example the notions of actual objects like (**High**, **Monday**). For that purpose we introduce for a given CSS the set of tuples of semantic concepts over M.

Definition 3. ***"Tuples of semantic concepts"***
Let $\mathfrak{K} := (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ *be a CSS. Then*

$$\mathcal{T}(M) := \{(\boldsymbol{c}_m)_{m\in M^*} \mid \boldsymbol{c}_m \in \mathfrak{B}(\mathbb{S}_m), \emptyset \neq M^* \subseteq M\}$$

is called the set of tuples of semantic concepts over M*.*

Remark: Of course a tuple $(\mathbf{c}_m)_{m\in M^*}$ is understood as a mapping which maps each element $m \in M^*$ to a formal concept $\mathbf{c}_m \in \mathfrak{B}(\mathbb{S}_m)$.

Definition 4. ***"Key of a CSS"***
Let $\mathfrak{K} := (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ *be a CSS. A set* $K \subseteq M$ *is called a key of* $\mathfrak{K}$*, if the mapping* $\lambda_K : G \longrightarrow \mathcal{T}(K)$ *where* $\lambda_K(g) := (\lambda(g, m))_{m\in K}$ *is injective.*

In the following we do not assume that there exists a key of a CSS $\mathfrak{K}$. Even the attribute set M need not be a key, that is "multiple rows" are not forbidden. Clearly, if there exists a key K of $\mathfrak{K}$, then the tuples of $\lambda_K(G)$, called the occurring K-tuples, could be used for the representation of "objects in reality" which could then be represented by the formal objects in G. That corresponds to the standard interpretation of formal objects in contexts and many-valued contexts.

The tuples of semantic concepts will be used in the following twice: first, we will use it in the next section in the definition of a *selection*. Second, we will introduce a specified subset $\mathcal{O}$ of $\mathcal{T}$(M) as the set of temporal objects in the definition of a Temporal Conceptual Semantic System. In our example of a moving high pressure zone the 1-tuple (**High**) is a temporal object. To define for a temporal object its *state at a time granule* in some space, for example in a weather map, as indicated in Fig. 3 by an ellipse with the label (**High**, **Monday**), we introduce in the next section the notion of a σ-Q-trace of a tuple where σ is a *selection* and Q is a *view*.

3.4 Views, selections, and traces

Now we recall the definitions of *views*, *selections*, and *traces* from [32].

Definition 5. ***"View"***
Let $\mathbb{K} := (G, N, J)$ *be a formal context. Then any subset* $Q \subseteq N$ *is called a* ***view*** *of* $\mathbb{K}$*. The subcontext* $\mathbb{K}_Q := (G, Q, J \cap (G \times Q))$ *is called the* $Q-$*part of* $\mathbb{K}$*.*

In the following the concept lattice of the $Q-$part of a view Q of the semantically derived context of a CSS will be used as a "landscape" into which further information is embedded. To describe this embedding of information we use the following notion of a *selection*.

Definition 6. ***"Instance selection"***
Let $\mathfrak{K} := (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ *be a CSS,* $\mathcal{T}(M)$ *the set of tuples of semantic concepts over* M*. For a subset* $\mathcal{T} \subseteq \mathcal{T}(M)$ *any mapping*

$$\sigma : \mathcal{T} \to \mathfrak{P}(G) := \{X \mid X \subseteq G\}$$

is called an instance selection on $\mathcal{T}$*. For* $\mathbf{c} \in \mathcal{T}$ *the set* $\sigma(\mathbf{c}) \subseteq G$ *is called the instance selection of* $\mathbf{c}$ *or the selection of* $\mathbf{c}$*.*

The standard instance selection in database theory is

$$\sigma_{db}((\mathbf{c}_m)_{m\in M^*}) := \{g \in G \mid \forall_{m\in M^*}\ m(g) = \mathbf{c}_m\}.$$

In Conceptual Semantic Systems we may use other instance selections, for example

$$\sigma_\alpha((\mathbf{c}_m)_{m\in M^*}) := \{g \in G \mid \forall_{m\in M^*}\ m(g) \leq \mathbf{c}_m\}.$$

The instance selection σ_α is used in our example to select for the formal concept **High** of the pressure scale all instances g such that the pressure value of g is a sub-concept of **High**. It is noteworthy that **High** does not occur as a pressure value in Tab. 1.

Definition 7. ***"σ-Q-trace of a tuple"***
Let $\mathfrak{K} := (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m \in M}, \lambda)$ *be a CSS with semantically derived context* $\mathbb{K} := (G, N, J)$*,* σ *an instance selection on* $\mathcal{T} \subseteq \mathcal{T}(M)$*,* $\boldsymbol{c} \in \mathcal{T}$*, and* $Q \subseteq N$ *a view with object concept mapping* γ_Q*. Then the set* $\gamma_Q(\sigma(\boldsymbol{c}))$ *is called the* σ*-*Q*-trace of the tuple* $\boldsymbol{c}$*.*

In our example of the moving high pressure zone the selection of the tuple $(\mathbf{High}, \mathbf{Monday})$ is chosen as the set of all instances g such that $pressure(g) \geq 1010$ and $time(g) = \mathbf{Monday}$. The σ-Q-trace of this tuple is marked in Fig. 3 by the left ellipse. That is conceptually the same technique as to represent towns, mountains, and rivers in a geographical map. In that sense, the notion of a trace generalizes also the geometrical notion of a volume.

3.5 Precise and distributed tuples

Definition 8. ***"precise and distributed tuples"***
Let $\mathfrak{K} := (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m \in M}, \lambda)$ *be a CSS with semantically derived context* $\mathbb{K} := (G, N, J)$*,* σ *an instance selection on* $\mathcal{T} \subseteq \mathcal{T}(M)$*,* $\boldsymbol{c} \in \mathcal{T}$*, and* $Q \subseteq N$ *a view with object concept mapping* γ_Q*. Then* $\boldsymbol{c}$ *is called*

- σ*-precise in* $\mathfrak{B}(\mathbb{K}_Q) \iff |\gamma_Q(\sigma(\boldsymbol{c}))| = 1$*;*
- σ*-distributed in* $\mathfrak{B}(\mathbb{K}_Q) \iff |\gamma_Q(\sigma(\boldsymbol{c}))| \geq 2$*.*

The tuple $(\mathbf{High}, \mathbf{Monday})$ is a good example for a *distributed* tuple since its σ-Q-trace has more than one object concept. Each of the towns in Fig. 3 has a trace with exactly one object concept. They are examples of *precise* 1-tuples.

4 Temporal Conceptual Semantic Systems

Temporal Concept Analysis has been introduced by the author [22, 23] as the theory of temporal phenomena described with tools of Formal Concept Analysis. The notion of a Temporal Conceptual Semantic System generalizes previous notions of temporal systems in Temporal Concept Analysis. It covers discrete, continuous, and hybrid systems as special cases.

From [31, 32] we recall the definition of a Temporal Conceptual Semantic System (TCSS). There are four main ideas which are mathematically described in the definition of a TCSS:
1. A TCSS should be a CSS having a specified many-valued *time attribute* T whose scale contains all temporal concepts which are needed for the given purpose. Instead of a single time attribute we could also introduce a non-empty set of time attributes. The corresponding changes are explicitly written down by the author in his paper [35] on Temporal Relational Semantic Systems.
2. It also should have a specified set $\mathcal{O}$ of *temporal objects* which are used to describe moving objects (like cars) as opposed to static objects (like houses), clearly with respect to the given purpose.
3. Each temporal object $\mathfrak{o} \in \mathcal{O}$ is associated with a binary relation $\mathcal{R}_{\mathfrak{o}}$ of *base transitions* where each base transition is a pair of formal concepts from the time scale $\mathbb{S}_T$. Each base transition of the temporal object $\mathfrak{o}$ describes one step of $\mathfrak{o}$ in time; other temporal objects may do other steps in time.
4. Each temporal object $\mathfrak{o}$ of a TCSS is represented as a tuple of semantic concepts over M:
$\mathfrak{o} := (\mathbf{c}_m)_{m \in M^*} \in \mathcal{T}(\mathrm{M})$.

Definition 9. ***"Temporal Conceptual Semantic System"***
Let $\mathfrak{K} := (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$ *be a CSS,* $T \in M$*,* $\mathcal{O} \subseteq \mathcal{T}(M)$ *, and for each* $\mathfrak{o} \in \mathcal{O}$ *let* $\mathcal{R}_{\mathfrak{o}}$ *be a binary relation on* $\mathfrak{B}(\mathbb{S}_T)$*. Then the quadruple*

$$(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_{\mathfrak{o}})_{\mathfrak{o}\in\mathcal{O}})$$

is called a Temporal Conceptual Semantic System (*TCSS*) *with time attribute* T*, the set* $\mathcal{O}$ *of temporal objects, and for each temporal object* $\mathfrak{o} \in \mathcal{O}$ *its time relation* $\mathcal{R}_{\mathfrak{o}}$*. The elements of* $\mathcal{R}_{\mathfrak{o}}$ *are called base transitions of* $\mathfrak{o}$*. The elements of* $\mathfrak{B}(\mathbb{S}_T)$ *are called time granules.*

One of the central notions in temporal systems is that of a state which is introduced in the following section.

4.1 States

The notion of a state is used in all system descriptions, but, to the best of my knowledge, nowhere defined mathematically clear in such a way that it is connected with a formal description of objects, time, and granularity [27, 36]. That seems to be necessary if we wish to say that "an object is at some time granule in some state". The author has introduced in [20] a definition of a state in a Conceptual Time System as an object concept of a formal object which was interpreted as a time granule. Later on this definition was extended to CTSOTs where pairs (p, t) of objects (e.g persons) and time granules are used as formal objects. That led to a very useful notion of a state of an object at some time granule, but that state was also just a single object concept. That did not fit with the idea that a state should be connected with a certain (probability) distribution, as for example in Quantum Theory [2, 15]. The introduction of CSSs and its distributed objects yields a solution also for this case [28].

The following definition generalizes the notion of a state as defined in [28] by introducing arbitrary instance selections and the object representation by tuples of semantic concepts.

Definition 10. ***"State of a temporal object at a time granule"***
Let $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_{\mathfrak{o}})_{\mathfrak{o}\in\mathcal{O}})$ *be a TCSS and* $\mathfrak{K} = (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$*. Let* σ *be an instance selection and* Q *be a view of the semantically derived context* $\mathbb{K} = (G, N, J)$ *of* $\mathfrak{K}$*. For each temporal object* $\mathfrak{o} = (\boldsymbol{c}_m)_{m\in M^*} \in \mathcal{O}$ *where* $T \notin M^*$ *and each time granule* $\boldsymbol{t} \in \mathfrak{B}(\mathbb{S}_T)$ *the tuple* $(\mathfrak{o}, \boldsymbol{t})$ *is called an "actual object". The* σ*-*Q*-state of the temporal object* $\mathfrak{o}$ *at time granule* $\boldsymbol{t}$ *is defined as*

$$\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))$$

which is the σ*-*Q*-trace of the actual object* $(\mathfrak{o}, \boldsymbol{t})$*.*

Remark: The actual object $(\mathfrak{o}, \mathbf{t})$ is an $(n + 1)$-tuple, if the temporal object $\mathfrak{o}$ is an n-tuple of scale concepts.

Examples:

1. If a TCSS $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_{\mathfrak{o}})_{\mathfrak{o}\in\mathcal{O}})$ has a many-valued attribute $P \neq T$ whose semantic scale is interpreted as a scale for persons, and $\{P, T\}$ forms a key in $\mathfrak{K}$, then the TCSS can be represented as a CTSOT [23, 27], and for each view Q of the semantically derived context $\mathbb{K}$ each actual person $(\mathbf{p}, \mathbf{t}) \in \mathfrak{B}(\mathbb{S}_P) \times \mathfrak{B}(\mathbb{S}_T)$ is precise in $\mathfrak{B}(\mathbb{K}_Q)$ with respect to the usual database selection σ_{db}. In that sense each person is at each time granule at exactly one "place" (= object concept) in $\mathfrak{B}(\mathbb{K}_Q)$. That led for CTSOTs to the notion of a *life track* of a person [23, 27]. The following definitions of *life spaces* and *life tracks* generalize that notion to TCSSs.

2. Distributed states: If the σ-Q-state of the temporal object $\mathfrak{o}$ at time granule $\mathbf{t}$ is distributed in the sense that the actual object $(\mathfrak{o}, \mathbf{t})$ is σ-distributed in $\mathfrak{B}(\mathbb{K}_Q)$, then the relative frequency distribution of the instances $g \in G$ on the object concepts of the σ-Q-state $\gamma_Q(\sigma(\mathfrak{o}, \mathbf{t}))$ forms a probability distribution associated with that state. As opposed to the probability distribution of a quantum mechanical state the probability distribution of a σ-Q-state has a clear frequency interpretation which is relevant for practical measurements. For the special case of the instance selection σ_α and a spatio-temporal CSS this has been discussed in detail by the author in [28].

4.2 Life space and life track

In this section we generalize the notion of life tracks as introduced in the framework of CTSOTs [27] to TCSSs.

Definition 11. ***"Life space and life track of a temporal object"***
Let $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_\mathfrak{o})_{\mathfrak{o}\in\mathcal{O}})$ be a TCSS and $\mathfrak{K} = (G, M, (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$. Let σ be an instance selection and Q be a view of the semantically derived context $\mathbb{K} = (G, N, J)$ of $\mathfrak{K}$. For each temporal object $\mathfrak{o} = (\boldsymbol{c}_m)_{m\in M^*} \in \mathcal{O}$ where $T \notin M^*$ we call the set

$$\mathcal{S}_{\sigma Q}(\mathfrak{o}) := \bigcup_{\boldsymbol{t}\in T(G)} \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))$$

the σ-Q-life space of the temporal object $\mathfrak{o}$.
The set

$$\mathcal{L}_{\sigma Q}(\mathfrak{o}) := \{((\mathfrak{o}, \boldsymbol{t}), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))) \mid \boldsymbol{t} \in T(G)\}$$

is called the labeled σ-Q-life space of $\mathfrak{o}$.
If $|\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))| \leq 1$ for each $\boldsymbol{t} \in T(G)$, then we call $\mathcal{L}_{\sigma Q}(\mathfrak{o})$ the σ-Q-life track of $\mathfrak{o}$.

Remarks:
(1) The notion of a life space of an object can be used for example for the formal representation of the habitat of a biological species from temporal occurrence data of individual animals of that species. In that case it would be useful to choose a semantic scale for the observed animals of that species where the single animals are sub-concepts of a species-concept. For that scale the instance selection σ_α will be useful for the generation of the habitat of that species in some geographic map $\mathfrak{B}(\mathbb{K}_Q)$.
(2) In our example of a moving high pressure zone (cf. Fig. 3) the labeled life space of the concept **High** is visualized by the two labeled ellipses.
(3) If the life space of a temporal object is a life track, then we get the usual labelling of a life track where each state is labeled by all time granules at which the object has "visited" that state.

4.3 Transitions

The basic idea of a transition of an object is a "change of the object from one state to another", for example the *transition of a person from one place to another*. As opposed to automata theory [1, 9] - where transitions are defined as pairs of states, and states are not explicitly connected with time, and objects are not made explicit - we use for the definition of a transition not only a pair of states, but also an object, which performs the transition, and its time relation.

To define a transition in a TCSS $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_\mathfrak{o})_{\mathfrak{o}\in\mathcal{O}})$ we use a temporal object $\mathfrak{o} \in \mathcal{O}$ as a representation of an object which performs the transition. To represent the direction from the initial place of the transition of $\mathfrak{o}$ to the final place of the transition we use a *base transition* $(\mathbf{s}, \mathbf{t}) \in \mathcal{R}_\mathfrak{o}$. To represent the space in which the transition is performed we use the concept lattice $\mathfrak{B}(\mathbb{K}_Q)$ of a suitable view Q of the semantically derived context of $\mathfrak{K}$. To represent the place of an object $\mathfrak{o}$ at a time granule $\mathbf{t}$ in the "map" $\mathfrak{B}(\mathbb{K}_Q)$ we use the σ-Q-trace of the actual object

$(\mathfrak{o}, \mathbf{t})$ with respect to some instance selection σ. The following definition of a σ-Q-transition of a temporal object generalizes the corresponding definition of a transition in a CTSOT [27].

Definition 12. ***"σ-Q-transition of a temporal object"***
Let $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_\mathfrak{o})_{\mathfrak{o}\in\mathcal{O}})$ be a TCSS and $\mathfrak{K} = (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$. Let σ be an instance selection and Q be a view of the semantically derived context $\mathbb{K} = (G, N, J)$ of $\mathfrak{K}$. For each temporal object $\mathfrak{o} = (\boldsymbol{c}_m)_{m\in M^*} \in \mathcal{O}$ where $T \notin M^*$ and each base transition $(\boldsymbol{s}, \boldsymbol{t}) \in \mathcal{R}_\mathfrak{o}$ we call the pair

$$(((\mathfrak{o}, \boldsymbol{s}), (\mathfrak{o}, \boldsymbol{t})), (\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{s})), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))))$$

the σ-Q-transition of $\mathfrak{o}$ induced by the base transition $(\boldsymbol{s}, \boldsymbol{t})$ leading from the initial place $((\mathfrak{o}, \boldsymbol{s}), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{s})))$ to the final place $((\mathfrak{o}, \boldsymbol{t}), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t})))$.

This definition of a transition has the advantage that the initial place as well as the final place of a transition are elements of the labeled life space of the given object.

4.4 The Life space digraph

On the labeled life space of an object $\mathfrak{o}$ we can easily introduce a digraph by "transporting" the time relation $\mathcal{R}_\mathfrak{o}$ into the life space. That generalizes the life track digraph for CTSOTs [27].

Definition 13. ***"Life space digraph of a temporal object"***
Let $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_\mathfrak{o})_{\mathfrak{o}\in\mathcal{O}})$ be a TCSS and $\mathfrak{K} = (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$. Let σ be an instance selection and Q be a view of the semantically derived context $\mathbb{K} = (G, N, J)$ of $\mathfrak{K}$. For each temporal object $\mathfrak{o} = (\boldsymbol{c}_m)_{m\in M^*} \in \mathcal{O}$ where $T \notin M^*$ the life space digraph of $\mathfrak{o}$ is defined as the directed graph

$$(\mathcal{L}_{\sigma Q}(\mathfrak{o}),\ \hat{\mathcal{R}}_\mathfrak{o})$$

where

$$((\mathfrak{o}, \boldsymbol{s}), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{s})))\ \ \hat{\mathcal{R}}_\mathfrak{o}\ \ ((\mathfrak{o}, \boldsymbol{t}), \gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))) \Leftrightarrow \boldsymbol{s}\ \ \mathcal{R}_\mathfrak{o}\ \ \boldsymbol{t}.$$

An example of a small life space digraph with two vertices and one arc can be seen in Fig. 3 where the thick arrow represents the σ-Q-transition of the object **High** induced by the base transition (**Monday**,**Tuesday**).

The life space digraphs can be used very effectively for animations of TCSSs. First computer versions for these animations exist in the computer program SIENA which is a part of the program TOSCANAJ [3, 4].

4.5 Particles and waves in Temporal Conceptual Semantic Systems

In [26] the author has introduced the notions of particles and waves in spatio-temporal Conceptual Semantic Systems based on the special instance selection σ_α and on a more restrictive notion for objects. In the following definition we generalize that to Temporal Conceptual Semantic Systems with its representation of objects by tuples of semantic concepts and with arbitrary instance selections.

Definition 14. ***"Particles and Waves"***
Let $(\mathfrak{K}, T, \mathcal{O}, (\mathcal{R}_\mathfrak{o})_{\mathfrak{o}\in\mathcal{O}})$ be a TCSS and $\mathfrak{K} = (G,\ M,\ (\underline{\mathfrak{B}}(\mathbb{S}_m))_{m\in M}, \lambda)$. Let σ be an instance selection and Q be a view of the semantically derived context $\mathbb{K} = (G, N, J)$ of $\mathfrak{K}$. A temporal object $\mathfrak{o}$ is called a

– *σ-Q-particle* $\Leftrightarrow |\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))| \leq 1$ *for all* $\boldsymbol{t} \in T(G)$*;*
– *σ-Q-wave* $\Leftrightarrow |\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t}))| \geq 2$ *for all* $\boldsymbol{t} \in T(G)$*;*
– *full σ-Q-wave* $\Leftrightarrow \mathfrak{o}$ *is a σ-Q-wave and* $\gamma_Q(\sigma(\mathfrak{o}, \boldsymbol{t})) = \gamma_Q(G)$ *for all* $\boldsymbol{t} \in T(G)$*.*

It has been proven by the author in [26] that each classical particle in physics can be described by a particle as defined above, and each classical wave can be described by a wave as defined above. Of course many more specific wave notions can be defined.

5 An Application of TCSS: The behavior of a distillation column

5.1 The data of the distillation column

The following application of Temporal Conceptual Semantic Systems in the chemical industry shows the behavior of a distillation column during 20 days. At each day the values of 13 many-valued attributes ("variables") like reflux, energy1, input, and pressure have been measured once. The resulting data table with 20 rows and 13 columns has the variable *day* as its key. This data table is one of the most simple types of a TCSS. There is only one temporal object, namely the distillation column. The time is represented just by the labels from 1 to 20 for the 20 days of measurement. The time relation for the single temporal object is just the predecessor relation on the integers from 1 to 20. All many-valued attributes have been scaled by an ordinal scale having a small chain as its concept lattice.

Even though the data table is quite small it is not easy to understand the behaviour of the distillation column in all 13 variables simultaneously. Therefore, we first studied the variables which are most important for the experts. It was easy for these experts to understand and interpret the behaviour of the distillation column in transition diagrams with two or three variables, like in Fig.4; but they had difficulties understanding not-nested line diagrams representing four or more variables.

5.2 Visualization of a life track in a nested line diagram

In the following we visualize the behaviour of the distillation column with respect to the four variables previously mentioned. Preparing this 4-dimensional representation we first show in Fig.4 the life track of the distillation column in a transition diagram for the variables *reflux* and *energy1*, each scaled with a 3-chain where the top concept represents the big values, the bottom concept the small values as indicated by the names of the scale variables, as for example "reflux ≤ 133".

We see in Fig.4 that the distillation column is at the first day in the state $\gamma(1)$, the object concept of day 1, labeled by 1. There the *reflux* is small and *energy1* lies in the middle category in this scaling. At the second day the distillation column is in the state $\gamma(2)$, the top concept in this lattice, where each of the two variables has a big value in this scaling. At the third day the destillation column "visits" the state of the first day again. The task to follow the life track visually can be supported by an animation using the computer program SIENA in TOSCANAJ [3, 4].

To visualize now the behavior of the destillation column for the four selected variables we first generate a concept lattice for the two variables *input* and *pressure* where the *input* is scaled in a 5-chain, while the *pressure* is scaled in a 4-chain in the same way as before. Using Peter Becker's latest (unpublished) 2010-version of SIENA one can generate from two given scales a nested line diagram. Employing the Temporal Concept Analysis tool of SIENA one can embed life tracks into the nested line diagram. This led to the nested transition diagram in Fig.5 which is explained now.

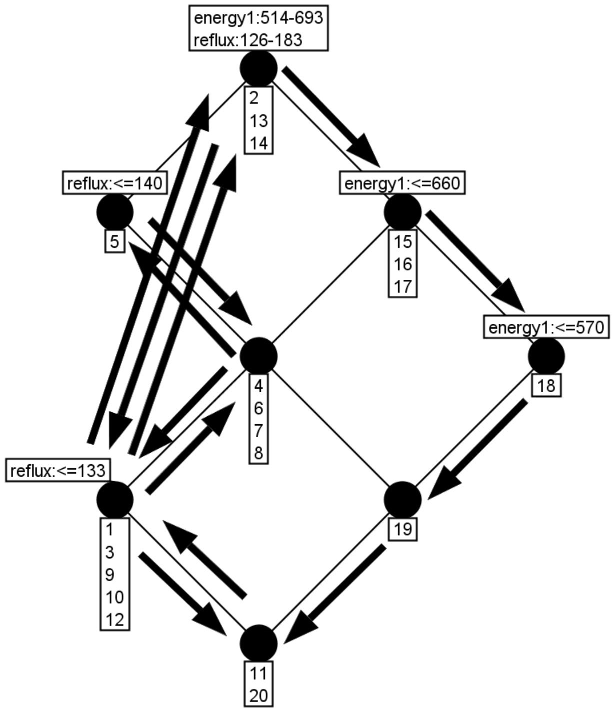

Fig. 4: Life track of a distillation column in a line diagram representing reflux and energy1

The coarse structure represented by the 8 ellipses shows the concept lattice for the variables *reflux* and *energy1* as in Fig.4. The fine structure inside the ellipses shows the concept lattice for the variables *input* and *pressure*. Only in the top ellipse the attributes of the fine structure are named. In Fig.5 we see in the coarse structure, that at the first (and the ninth) day the measured value for the *reflux* is ≤ 133, for the *energy1* ≤ 660 and not ≤ 570 as we can see also from Fig.4. Looking at the fine structure in Fig.5 we see that at the first day the values for *input* are ≤ 630 and not ≤ 615 while the first-day values for the *pressure* are ≤ 120 and not ≤ 115. That shows the successful visualization of four variables in each single state of the distillation column. The meaning of the transitions between states is also successfully visualized: as a simple example we just mention, that arrows within a single ellipse represent transitions where the attributes of the coarser structure are constant in the granularity of the outer scale; for example the transition from the state at day 16 to the state at day 17, shortly described by the base transition (16,17), is constant in the outer scale, and changes only the value of the *input* from the top level to the next lower level. Another extrem example is the transition (1,2) which changes all four variables. In the same way the meaning of the other arrows can be easily understood from Fig.5.

The main advantage of the transition diagrams in this application was that the experts of this distillation column realized that they had some wrong ideas about the behaviour of their distillation column. Using transition diagrams in a self-chosen granularity the experts developed visually supported state spaces for the understanding and the control of the production processes in the distillation column.

For another useful application of TCSSs in a biomedical study of disease processes in arthritic patients the reader is referred to the next article in this volume: *Conceptual Representation of*

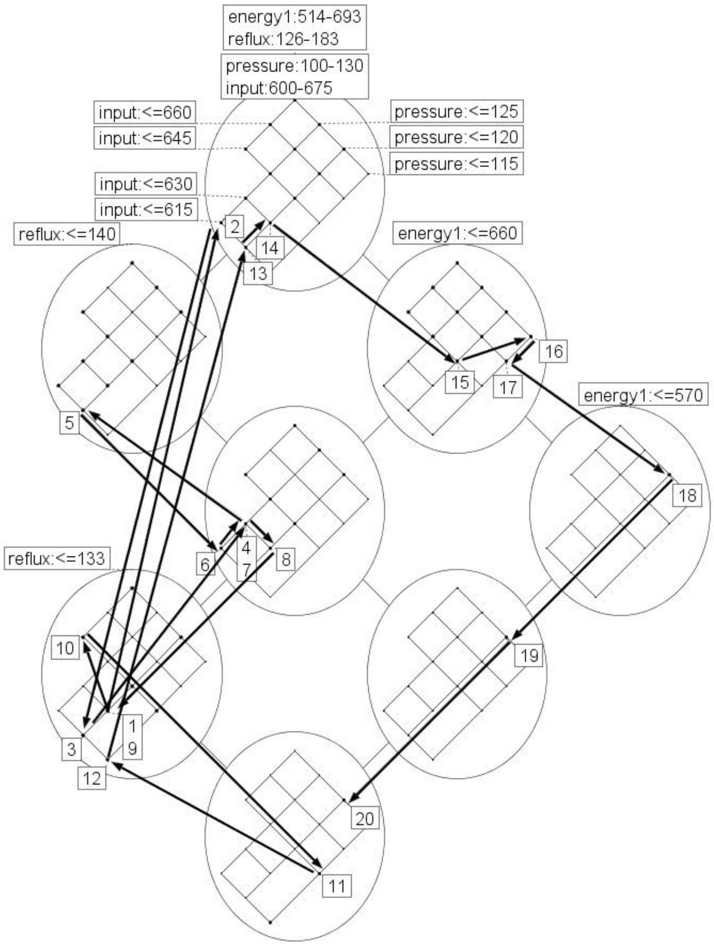

Fig. 5: Life track of a distillation column in a nested line diagram representing reflux, energy1, input and pressure

Gene Expression Processes by Johannes Wollbold, René Huber, Raimund Kinne, and Karl Erich Wolff.

6 Conclusions and Future Research

We have generalized the basic notions in Temporal Concept Analysis, namely states, transitions, life tracks, the digraph of a life track, particles and waves to the actual notion of a Temporal Conceptual Semantic System. The main new tools are the representation of objects by tuples of concepts of the semantic scales and the introduction of an instance selection which generalizes the usual instance selection for data bases. As opposed to classical time representations we did not use any linear ordering of the set of time granules, but all the classical time representations as continuous time, discrete time, and the usual time representation with several time attributes for years, months, and days with its cyclic time structures are included as special cases. One of the most promising advantages of Temporal Conceptual Semantic Systems is that the applicability is now much broader than before, since it is no longer necessary to search for an object domain which can be used as a key for the data. The main advantage with respect to the visualization is that trace diagrams combine the ordinal structure of concept lattice with the benefits of Venn diagrams.

Future research will focus on applications in many fields, mainly in the representation of relational data. A first success was the combination of conceptual scaling and concept graphs in Conceptual Relational Semantic Systems [33, 34] and its extension to Temporal Relational Semantic Systems [35]. For practical applications it will be very useful to introduce new graphical tools for the construction of animated trace diagrams into the computer programs of Formal Concept Analysis.

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